Home work 5

Question 1:

(1) Prob [A successful] =

= Prob [A transmits AND B does not].

= Prob [A transmits] . Prob [B does not]

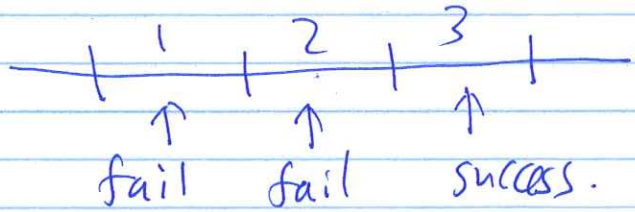
= 0.4 . (1 − 0.3)

= 0.4 . 0.7

= 0.28

(2) P_A = Prob [A successful in a slot]. (given).

Prob [A succeeds for 1st time in slot 3]:



= Prob [A fails is slot 1 ∧
A fails in slot 2 ∧
A succeeds in slot 3]

= Prob [A fails in slot 1] .
Prob [A fails in slot 2] .
Prob [A succeeds in slot 3]

= $(1 - P_A) . (1 - P_A) . P_A$.

(3) Prob [B successful]

= Prob [B transmits ∧ A does not]

= Prob [B transmits] · Prob [A does not]

= 0.3 · (1 − 0.4).

= 0.3 · 0.6

= 0.18.

(4). Fraction of slots with successful transmission

= Prob [a slot contains a successful transm]

= Prob [A succeeds OR B succeeds].

= Prob [A successful] + Prob [B successful]

= 0.28 + 0.18.

= 0.46.

(2) Given:

Arrival rate = 20 msgs/sec.

Message length = 0.1 sec.

G = arrival rate / message length.

time unit = 1 message!!!

Convert arrival rate to correct time unit:

$$\text{Arrival rate} = 20 \text{ msgs/sec}$$

$$= \frac{20 \text{ msgs}}{1 \text{ sec}} \times \frac{0.1}{0.1}$$

$$= \frac{2 \text{ msgs}}{0.1 \text{ sec}}$$

$$= \frac{2 \text{ msgs}}{1 \text{ message length}}$$

$\Rightarrow$ $G = 2$ msgs / time unit.

Now we can use the formulas.

(a) Throughput of unslotted Aloha:

$$S = G \cdot e^{-2G} \qquad G = 2 \text{ msgs/time unit}$$

$$= 2 \cdot e^{-2.2}$$

$$= \boxed{2 \cdot e^{-4}}$$

$$= 0.037$$

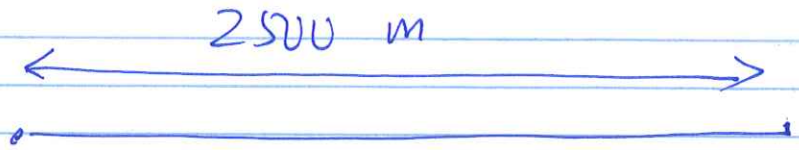
(b) Throughput of slotted Aloha:

$$S = G \cdot e^{-G} \qquad G = 2 \text{ msgs/time unit}$$

$$= \boxed{2 \cdot e^{-2}}$$

$$= 0.27$$

Question 3:



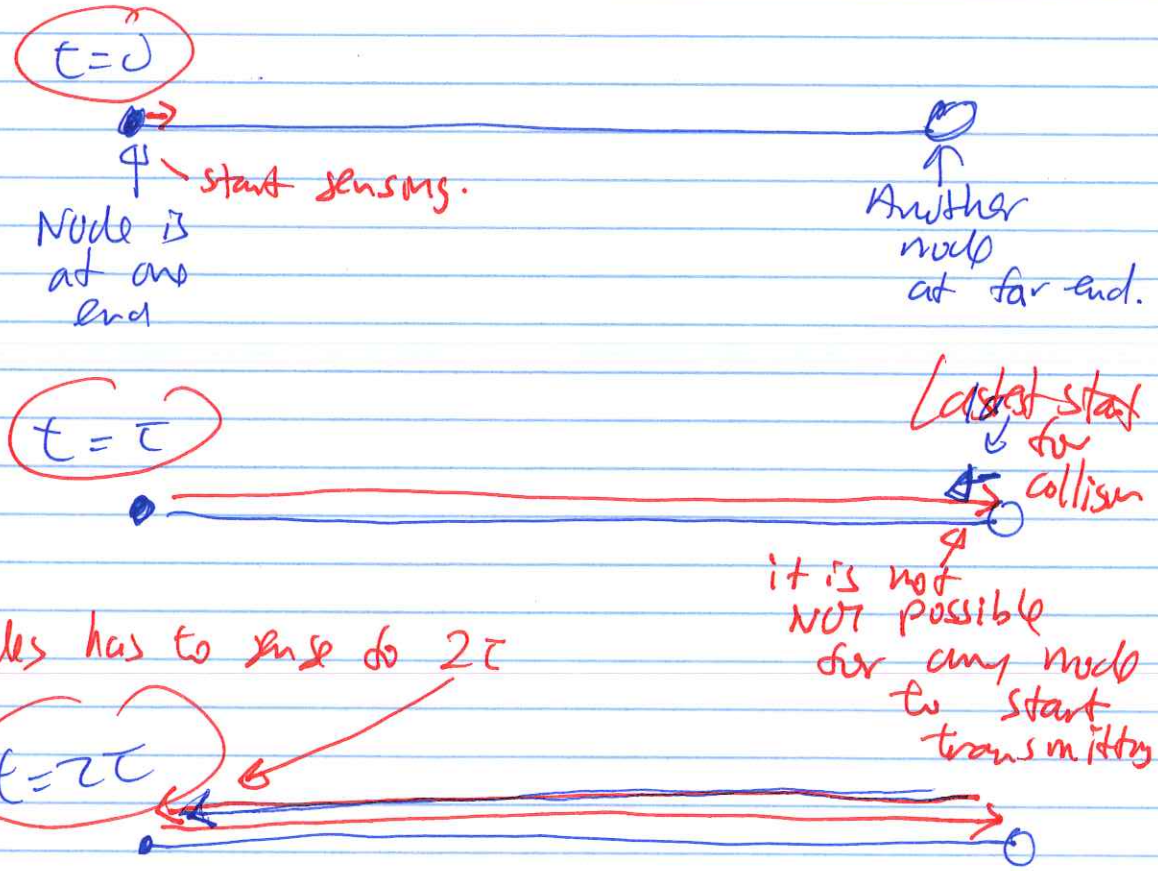
(1) $\tau = \frac{\text{distance}}{\text{speed}} = \frac{2500\ m}{2.10^8\ m/sec}$

$= 1.25\ 10^{-4}\ sec.$

(2). Node can be certain after 2τ sec.

$= 2.5\ 10^{-4}\ sec$

Worst case:



Question 4:

(1) A wins if A picks a smaller number than B.

A picks from 0, 1
B picks from 0, 1, 2, 3.

A wins if:

A picks	∧	B picks	Probability
0		1	$\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$
0		2	$\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$
0		3	$\frac{1}{8}$
1		2	$\frac{1}{8}$
1		3	$\frac{1}{8}$

$$\text{Prob}[\text{A wins}] = 5 \cdot \frac{1}{8} = \frac{5}{8}$$

(2) There is a collision if

A and B picks the same number.

There is a collision if:

A picks	∧	B picks	Probability
0		0	$\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$
1		1	$\frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$

$$\text{Prob}[\text{collision}] = 2 \cdot \frac{1}{8} = \frac{1}{4}$$

(3) A wins if A picks a smaller number than B.

A picks from 0, 1
B picks from 0, 1, 2, 3, 4, 5, 6, 7.

A wins if:

A picks ∧	B picks	Probability
0	1	each $\frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16}$.
0	2	
0	3	
0	4	
0	5	
0	6	
0	7	
1	2	each $\frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16}$.
1	3	
1	4	
1	5	
1	6	
1	7	

13 values

$$\text{Prob}[A \text{ wins}] = 13 \times \frac{1}{16} = \frac{13}{16}$$

(4) Probability of collision:

A picks ∧	B picks	Probability
0	0	each: $\frac{1}{2} \cdot \frac{1}{8} = \frac{1}{16}$
1	1	

$$\text{Prob}[\text{collision}] = 2 \times \frac{1}{16} = \frac{1}{8}$$