Intro to "twos complement encoding"

- Let's start with a small "Odometer code" using binary numbers:
  - 3 **binary** digits odometer
  - The odometer encoding is:

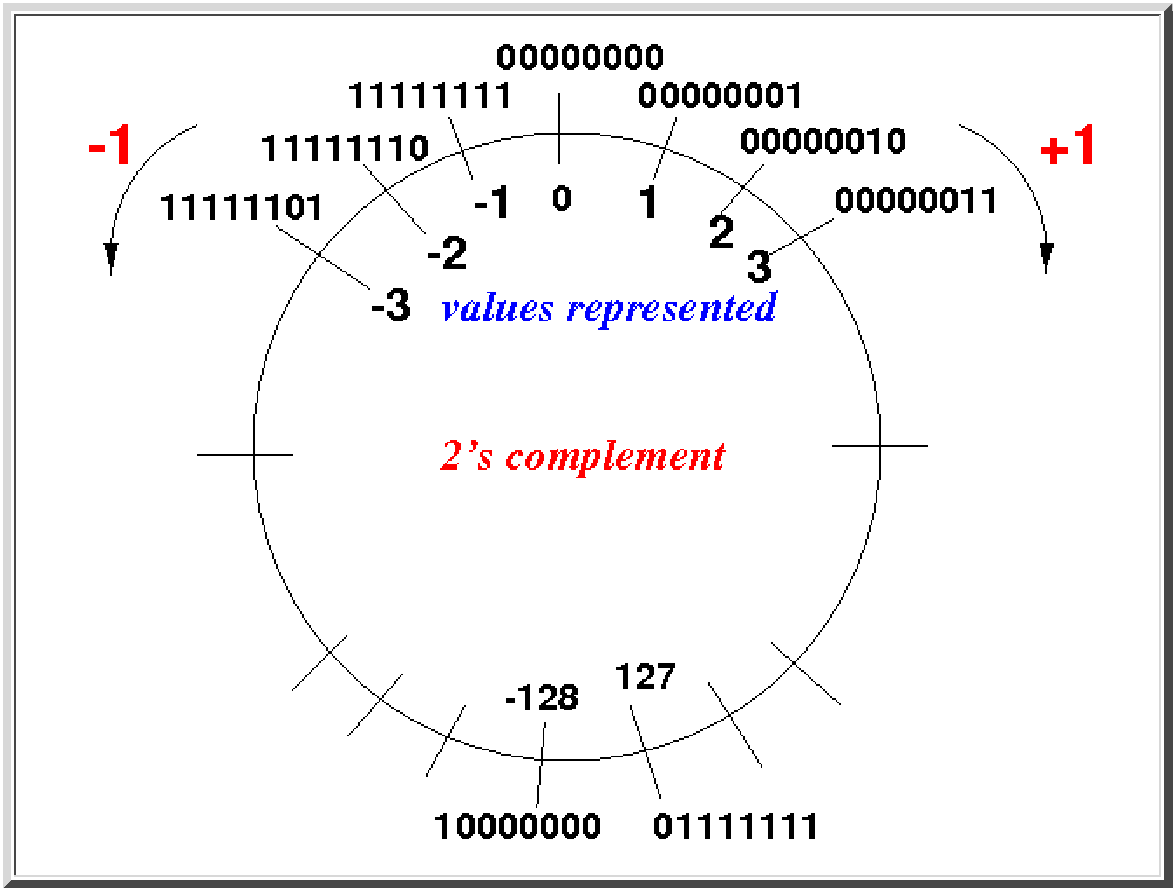
```
Odometer reading:   100  101  110  111  000  001  010  011
-----------------    +----+----+----+----+----+----+----+
Value represented:  -4   -3   -2   -1    0    1    2    3
```

- Note:
  - With 3 bits, we can represent **only** values between [-4, 3]
  - With 3 bits, we can values between $[-2^2, 2^2-1]$

- Let's look at "odometer code" using one byte of mamory:
  - 8 **binary** digits odometer
  - With 8 bits, we can values between $[-2^7, 2^7-1] = [-128, 127]$
  - The 2's complement number encoding is:

```
Code      Value
================
10000000  -128  <--- smallest negative value with 8 bits (-2^7)
10000001  -127
.....
11111000    -8
11111001    -7
11111010    -6
11111011    -5
11111100    -4
11111101    -3
11111110    -2
11111111    -1
00000000     0
00000001     1
00000010     2
00000011     3
00000100     4
00000101     5
00000110     6
00000111     7
00001000     8
.....
01111111   127  <--- largest positive value with 8 bits (2^7-1)
```

The **mapping** of the **representation** to the **value** that it represents is based on the following **circular (modulo) addition**:



Property:

- If you move **clock-wise** on **representation wheel**, you will **add 1** to the **representation**
  - Therefore, the **representation** that you get by **moving clock-wise** must be **1 larger** in **value**

- If you move ***counter* clock-wise** on **representation wheel**, you will **subtract 1** to the **representation**
  - Therefore, the **representation** that you get by **moving *counter* clock-wise** must be **1 larger** in **value**

- **Decoding a 2's complement representation**

  Again, to use 2s complement code, you need to know how to convert a value to 2s complement and vice versa

  - Look at the following table carefully to discover the coding & decoding method:

```
2's compl    Value       Compare with:  Value    Binary number system
================                        ============================
10000000     -128                         128     10000000
10000001     -127                         127     01111111
.....
11111000       -8                           8     00001000
11111001       -7                           7     00000111
11111010       -6                           6     00000110
```

```
11111011      -5                      5     00000101
11111100      -4                      4     00000100
11111101      -3                      3     00000011
11111110      -2                      2     00000010
11111111      -1                      1     00000001
00000000       0
00000001       1                      1     00000001
00000010       2                      2     00000010
00000011       3                      3     00000011
00000100       4                      4     00000100
00000101       5                      5     00000101
00000110       6                      6     00000110
00000111       7                      7     00000111
00001000       8                      8     00001000
.....
01111111     127                    127     01111111
```

Notice that:

- 2s complement representation for **positive** values is same as that used in binary number system

  Example:

  ```
       00000000
       00000001       1
       00000010       2
       00000011       3
       00000100       4
       00000101       5 <-----

  Representation
  ---------------
  00000101          ->    4 + 1 = 5
  ^    ^ ^
       | |
       4 1
  ```

- 2s complement representation for **negative** values added to the binary number for its absolute value is equal to 100000000.

  Example:

  ```
       11111010      -6
       11111011      -5 <-----
       11111100      -4
       11111101      -3
       11111110      -2
       11111111      -1
       00000000       0

       11111011    = representation for -5
     + 00000101    = representation for 5 (absolute value of -5)
    -------------
      100000000
  ```

These observations will help you understand the conversion procedures below.

- Converting a value **v** to its 2's complement code:
  - If value **v** is positive, then:
    - the 2's complement code is the same as its representation in the binary number system
  - If value **v** is negative, then:
    - First, obtain the binary number representation **x** for **-v** (note: **-v** is positive !)
    - Then, compute 100....000 **-** **x**, where the number 100....000 has exactly the same number of 0's as the number of bits in **x**.

  ```
  Example:
  ```

```
v = 7     The value is positive, so:
          (1) Binary number representation is: 111

          (2) 8 digit 2's complement representation is: 00000111
              16 digit 2's complement representation is: 0000000000000111
              and so on...

v = -7    The value is negative, so:
          (1) Binary number representation for 7 is: 111

          (2a) 8 digit 2's complement representation for 7 is: 00000111

          (3a) 8 digit 2's complement representation for -7 is:

                              100000000
                            -  00000111
                            -----------
                               11111001


          (2b) 16 digit 2's complement repr. for 7 is: 0000000000000111

          (3b) 8 digit 2's complement repr. for -7 is:

                              10000000000000000
                            -  0000000000000111
                            -----------
                               1111111111111001
```

- Converting a 2's complement code **c** to a signed value
  - If the encoding **c** begins with 0, it encodes a **positive** value and the value is "face value" in **binary** (but you will still need to convert it to decimal to be "comprehended" by humans)
  - If the encoding **c** begins with 1, it encodes a **negative** value and its absolute value is equal to 100...000 **- c** in **binary** (which again you will need to convert it to decimal to be "comprehended" by humans)

```
Example:

 code c = 00010010 -> it is a positive number
                      the value = 00010010 in binary

                      Convert to decimal:  0  0  0  1  0  0  1  0
                                                   16  +     2     = 18
                      Value = 18

 code c = 11101110 -> it is a negative number...

                      (1) Compute:      100000000
                                      -  11101110
                                      -----------
                                         00010010

                      (2) the absolute value of the negative value
                          is equal to 00010010 (binary), which is equal
                          to 18 (decimal)

                      (3) Since the value is negative, the value represented
                          by 11101110 is: -18 !
```

- **NOTE:** from the examples above:
  - 00000111 represents 7
    11111001 represents -7
  - 00010010 represents 18
    11101110 represents -18

that:

to **negate** a value, you must subtract the representation by 1000...000

- **NOTE:** there is an easier way to negate a 2s complement code:

```
To negate 7, we subtract the binary number 7 from 1000...0000

Example in 8 bits:

    100000000
   - 00000111 (= 7)
  -----------
     11111001 (= -7)
```

  The **subtraction** can be broken up in 2 steps as follows:

```
100000000 - 00000111 = (1 + 11111111) - 00000111
                     = 1 + (11111111 - 00000111)     [easy subtraction !]
                     = 1 + 11111000                  [result is same as flipping bits !]
```

  Summary: to negate a 2s complement representation:

  - Flip every bit in the 2s complement representation
  - Add 1 to the result

  Another example:

```
        As you saw above:    00010010 represents +18

        To get the representation for -18, you can do this:


        (1) Flip each bit:      00010010 -> 11101101

        (2) Add 1 to result:    11101101
                              +        1
                              ----------
                                11101110

   which is - as you saw above - the representation for -18
```

- Properties of 2's complement encoding:

  - Only one representation for ZERO (check for yourself)
  - Operations are "natural" - see examples below

- **Arithmetic with 2's complement encoding**

  - **Adding** 2's complement numbers:

```
               Values        8 digit 2's compl repr
Adding 2
positive           5                 00000101
values          +  9               + 00001001
                -----              ----------
                  14                 00001110  -> 8 + 4 + 2 = 14

Adding
positive +         5                 00000101
negative        + -9               + 11110111
                -----              ----------
                  -4                 11111100 -> represents -4
```

```
Adding
negative +        -5                11111011
positive        +  9              + 00001001
                -----             ----------
                   4                00000100 -> represents 4

Adding 2
negative          -5                11111011
values          + -9              + 11110111
                -----             ----------
                 -14                11110010 -> represents -14
```

- **Subtracting** 2's complement numbers:

```
               Values        8 digit 2's compl repr
Subtract 2
positive           5                00000101
values          -  9              - 00001001
                -----             ----------
                  -4                11111100 -> represents -4

Subtract
positive -         5                00000101
negative        - -9              - 11110111
                -----             ----------
                  14                00001110 -> represents 14


Subtract
negative -        -5                11111011
positive        -  9              - 00001001
                -----             ----------
                 -14                11110010 -> represents -14

Subtract 2
negative          -5                11111011
values          - -9              - 11110111
                -----             ----------
                   4                00000100 -> represents 4
```

- **Overflow**

Overflow

- What is "overflow":
  - Using 8 bits, we can represent the binary values between -128 and 127
  - Computer operation manipulate (change) the **representation**

    For example:

```
    00000011   <---- representation for the value *** (3)
  + 00000101   <---- representation for the value ***** (5)
 ------------
    00001000   <---- representation for the value ******** (8)
```

  - When the result of some operation on **byte** representations **falls outside** this range, then the **value** that is represented by the result is **different** from the correct value.
  - This phenomenon is called **overflow**
  - Overflow is a part of our daily life now that the computer is an integral part of our society and you should be aware of this phenomenon so you do not get caught by surprise...

- Here is a program that demonstrates the overflow phenomenon: click here **DEMO**
  - Try entering 1 + 1
  - and then: 127 + 1 (this will cause an overflow)
  - Do you understand why there is overflow at 127 using **byte** variables ??
- The following program is the same as the previous one, except I have added a function to print out the **binary representation** of the values.

  You can use this program to see why the program prints certain results: click here **DEMO**
- Computer can manipluate integers of various lengths:
  - **8 bits** (**byte type** in Java, char type in C, C++)
  - **16 bits** (**short type** in Java, C and C++)
  - **32 bits** (**int type** in Java, C and C++)
  - **64 bits** (**long type** in Java, C and C++)
  - **128 bits** (**long long type** in C and C++)
  - This program shows the effect of using more bits: click here **DEMO**
- Other types of variables also have **overflow problems**, just later...
  - **short** type variables will overflow at around 32000 (2^{15})
  - **int** type variables overflow at around 2 billion (2^{31})
  - Use the previous demo program to verify.

- **Converting between 2's complement representation of *different sizes***
  - The computer can use different numbers of bits to represent **signed integer** quantities:
    - byte (very short integer, values between -127 and 128)
    - short integer (values between -32767 and 32768)
    - (ordinary) integer (values between -2^{31} and 2^{31} - 1)
    - long integer (values between -2^{63} and 2^{63} - 1)
  - Sometimes, the programmer needs to convert a byte to a short or a short to an integer in the program.
  - This kind of operations is called a **type conversion**

    A **data type** is a certain data representation used in the computer

    The various kinds of integer representations (byte, short, int and long) are considered as **different data representations**
  - When the computer computer needs to convert (change) from one **represention** to another representation, the key of the change must be that: **the value of BOTH representation MUST BE EQUAL** (because the **value is intrinsic** and does not change)

- **Converting from a shorter representation to a longer representation**
  - **Sign extension** (widening conversion):
    - Notice that:
      - 8 bit 2s compl. repr. for 7 is: 00000111

- 16 bit 2s compl. repr. for 7 is: 0000000000000111
- 8 bit 2s compl. repr. for -7 is: 11111001
- 16 bit 2s compl. repr. for -7 is: 1111111111111001

- To obtain the **representation** for the **same** value using more bits, the computer must "extend" the left most bit.

- **Example**

```
int i;
short s;

s = 9;    <---- s is assigned 0000000000001001
i = s;    <---- assign an 16 bit integer to a 32 bit integer
                (1) 0000000000001001 is sign-extended to:
                    00000000000000000000000000001001
                (2) Then the value is store in variable i

s = -9;   <---- s is assigned 1111111111110111
i = s;    <---- assign an 16 bit integer to a 32 bit integer
                (1) 1111111111110111 is sign-extended to:
                    11111111111111111111111111110111
                (2) Then the value is store in variable i
```

- The **left most bit** in a **2's complement code** is a ***sign* bit**:

  - **All** of the **positive (and 0) values** are **represented** by **2's complement codes** that ***starts*** with `0.......`
  - **All** of the **negative values** are **represented** by **2's complement codes** that ***starts*** with `0.......`

- **Therefore**, we call this **"extend" the left bit** operation:

  - **Sign extension**

- **Converting from a longer representation to a shorter representation**

  - Narrowing conversion (casting):

    - Narrowing conversion is when you convert a value from a "longer" representation to a "shorter" representation.

      - You **truncate** the **left-most portion** of the **longer representation** to obtain the **shorter representation** of the **same value**

        But: you may **not** obtain a ***correct* representation** due to **overflow** !!!

    - **Example:**

```
int i;
short s;

i = 9;    <---- i is assigned 00000000000000000000000000001001
s = i;    <---- assign an 32 bit integer to a 16 bit integer
                (1) 00000000000000000000000000001001 is
```

```
                              truncated to 0000000000001001
                          (2) Then the value is store in variable s

      i = -9;  <---- i is assigned 11111111111111111111111111110111
      s = i;   <---- assign an 32 bit integer to a 16 bit integer
                          (1) 11111111111111111111111111110111 is
                              truncated to 1111111111110111
                          (2) Then the value is store in variable i
```

- Narrowing conversion (truncation) can result in a represention for a value that is **different than the original value**:

```
      i = 90000;   i is assigned 00000000000000010101111110010000
      s = i;       (1) 00000000000000010101111110010000 is truncated to
                       0101111110010000
                   (2) assigned to s
                       Problem: s represents 24464, (some bits lost !)
```

- Program showing narrowing conversion: click here **DEMO**

- The following program showing what happens when you convert:
  - byte -> short or int
  - short -> byte or int
  - int -> byte or short

  Get the program here: click here **DEMO**

  - There are no problems from byte -> short or int
  - Try entering 89 and (restart program) 1000 as a short, you will see overflow in the byte variable
  - Try entering 89, (restart program) 1000 and (restart again) 80000 as int, you will see overflow in the byte variable for 1000, and overflow in short and byte for 80000.
