Multiplication with Base-5 Numbers

- **Base-5 number system**: number system based on number 5
  - Has **5** digits: 0, 1, 2, 3, 4
  - Value of digits increase by a factor of **5** for each position

```
Example:

   243(5) = 2 x 5^2 + 4 x 5^1 + 3 x 5^0
          = 2 x 25 + 4 x 5 + 3 x 1
          = 50 + 20 + 3 = 73
```

- **Base-5 addition**
  - You must remember that the digit 0 follows the digit 4 in the base-5 system
  - So:

```
1 + 1 =  2     1 + 2 =  3     1 + 3 =  4     1 + 4 = 10
2 + 1 =  3     2 + 2 =  4     2 + 3 = 10     2 + 4 = 11
3 + 1 =  4     3 + 2 = 10     3 + 3 = 11     3 + 4 = 12
4 + 1 = 10     4 + 2 = 11     4 + 3 = 12     4 + 4 = 13
```

  - **More complex examles...**

Example 1:

```
   12
    2
-------
   14
```

Example 2:

```
   12
    3
-------
   20          2 + 3 = 10, write down 0 and remember carry 1
               1 + carry 1 = 2
```

Example 3:

```
   141
  2340
 -----
  3031                        1 + 0 = 1
                          4 + 4 = 13, write down 3, remember carry 1
                          1 + 3 = 4, add carry 1 = 10, write down 0, carry 1
                          2 + carry 1 = 3
```

- **Base-5 multiplication**
  - In order to perform multiplication in base-5, we need to "memorize" the multiplication table for base-5.
  - The following is the multiplcation table for base-5:

```
    |  1   2   3   4
----+----------------
  1 |  1   2   3   4
  2 |  2   4  11  13
```

```
3 |  3  11  14  22
4 |  4  13  22  31
```

- 2x3 = 11, because:

```
2 x 3 = value 6 ----- which is represented in base-5 as 11  (1 x 5 + 1 = 6)
```

- 2x4 = 13, because:

```
2 x 4 = value 8 ----- which is represented in base-5 as 13  (1 x 5 + 3 = 8)
```

- 3x3 = 14, because:

```
3 x 3 = value 9 ----- which is represented in base-5 as 14  (1 x 5 + 4 = 9)
```

- 3x4 = 22, because:

```
3 x 4 = value 12 ----- which is represented in base-5 as 22  (2 x 5 + 2 = 12)
```

- 4x4 = 31, because:

```
4 x 4 = value 16 ----- which is represented in base-5 as 31  (3 x 5 + 1 = 16)
```

- We can now perform base-5 multiplication using the above table.

Example 1:

```
Base-5         Value

  23            2 x 5 + 3 = 13
   4                      = 4
 ----
  202           4 x 3 = 22, write down 2 and remember carry 2
                4 x 2 = 13, add the carry 2:
                            13 + 2 = 20
                            write down 0 and remember carry 2
```

In other words: $23_{(5)} \times 4_{(5)} = 202_{(5)}$

We can check the correctness:

$$202_{(5)} = 2 \times 5^2 + 0 \times 5^1 + 2 \times 5^0$$
$$= 2 \times 25 + 2$$
$$= 52 \text{ (which is equal to } 13 \times 4\text{)}$$

Example 2:

```
Base-5         Value

  43            4 x 5 + 3 = 23
  32            3 x 5 + 2 = 17
 ----
  141
 2340
-----
 3031
```

In other words: $43_{(5)} \times 32_{(5)} = 3031_{(5)}$

We can check the correctness:

$$3021_{(5)} = 3 \times 5^3 + 0 \times 5^2 + 3 \times 5^1 + 1 \times 5^0$$
$$= 3 \times 125 + 0 \times 25 + 3 \times 5 + 1$$
$$= 375 + 15 + 1$$
$$= 391 \text{ (which is equal to } 23 \times 17\text{)}$$