

2023 - ISU Putnam Practice Set 5

Friday, October 4, 2024

Probability

1. Imagine that you have an urn filled with 100 balls, some red and some green. You can't see inside; all you know is that someone determined the number of red balls by picking a number between zero and 100 from a hat. You reach into the urn and pull out a ball. It's red. If you now pull out a second ball, is it more likely to be red or green (or are the two colors equally likely)?
2. There are two jet planes and a propeller plane at the small regional airport of Gauss City. A plane departs from Gauss City and arrives in Eulerville, where there were already five propeller planes and one jet plane. Later, a farmer sees a jet plane flying out of Eulerville. What is the probability that the plane that arrived from Gauss City was a propeller plane, provided that all events are equiprobable?
3. If a needle of length 1 is dropped at random on a surface ruled with parallel lines at distance 2 apart, what is the probability that the needle will cross one of the lines?
4. Let C be the unit circle $x^2 + y^2 = 1$. A point p is chosen randomly on the circumference C and another point q is chosen randomly from the interior of C (these points are chosen independently and uniformly over their domains). Let R be the rectangle with sides parallel to the x and y -axes with diagonal pq . What is the probability that no point of R lies outside of C ?
5. Four points are chosen at random on the surface of a sphere. What is the probability that the center of the sphere lies inside the tetrahedron whose vertices are at the four points? (It is understood that each point is independently chosen relative to a uniform distribution on the sphere.)