Linear Algebra Exercises

Look at the exercises below (from Sasha) and try to solve as many as possible before our meeting. Note: I_n is the $n \times n$ identity matrix.

1. A Hadamard Matrix is an $n \times n$ matrix such that every entry is 1 or -1 and every pair of columns is orthogonal. Prove that if A is a Hadamard matrix and $n > 2$ then n is a multiple of 4.

2. Do there exist $n \times n$ matrices A, B such that $AB - BA = I_n$?

3. Let $A = (a_{ij})$ be an $n \times n$ matrix such that

$$\sum_{j=1}^{n} |a_{ij}| < 1$$

for all i. Prove that $I_n - A$ is invertible.

4. Prove that in $\mathbb{R}^n$ there is no set of $n + 2$ vectors whose pairwise angles are all obtuse.