Abstract: This dissertation is about arithmetic information encoded by analytic characteristics (such as Fourier coefficients) of classical modular forms and a real-analytic generalization of modular forms called harmonic Maass forms. For example, I use the theory of harmonic Maass forms to extend and refine a theorem of Wiles on class number divisibility. I also prove asymptotic bounds for Rankin-Selberg shifted convolution L-functions in shift aspect. In partition theory, I use the circle method to describe the expected distribution of parts of integer partitions over residue classes, and I use effective estimates for partition functions to obtain simple formulas for functions arising in group theory.